

Review Session Handout #14

Chapters 16–18: Asymmetric Information; Auctions and Bidding Strategies;
Externalities, Public Goods, and Common Resources

Introduction to Microeconomics: Markets, Strategy, and Real-World Cases

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Exercise R14.1. A used-car market has 100 cars. Half are good and half are lemons. A good car is worth \$12,000 to its owner and \$15,000 to a buyer; a lemon is worth \$4,000 to its owner and \$6,000 to a buyer. (These values differ from those of the example in Section 16.2.) Each owner knows the quality of her own car, but buyers cannot tell good cars from lemons.

- (a) If buyers know that half of the cars are good, what is the most they are willing to pay for a car?
- (b) At that price, which owners are willing to sell? Which owners drop out?
- (c) If only lemons remain on the market, what do buyers now offer?
- (d) Explain how adverse selection unravels the market for good cars.
- (e) Name two real-world mechanisms that reduce adverse selection in used-car markets.

Solution.

Part (a). A buyer who cannot tell the two kinds of car apart pays at most the average value to a buyer (Section 16.2):

$$\frac{1}{2} \times \$15,000 + \frac{1}{2} \times \$6,000 = \$7,500 + \$3,000 = \$10,500.$$

The most a buyer is willing to pay is \$10,500.

Part (b). At a price of \$10,500:

- Owners of lemons value their cars at \$4,000, less than \$10,500, so they are willing to sell.
- Owners of good cars value their cars at \$12,000, more than \$10,500, so they refuse to sell and drop out of the market.

Only owners of lemons remain.

Part (c). Buyers realize that any car offered at \$10,500 must be a lemon, which is worth \$6,000 to them, so they offer at most \$6,000. Owners of lemons, who value their cars at \$4,000, are still willing to sell at this price, so only lemons trade, at \$6,000.

Part (d). Because buyers cannot observe quality, they pay at most the average value of the cars they expect to be offered. That average is too low for owners of good cars, who withdraw. The cars left for sale are worse on average, so buyers lower what they will pay, until only lemons trade. Every good car would be worth \$3,000 more to a buyer than to its owner (\$15,000 – \$12,000), so the 50 good cars that never trade

represent \$150,000 of lost gains from trade. This is the lemons problem of Akerlof (1970): the hidden quality is correlated with who chooses to sell, so the bad drives out the good.

Part (e). Two mechanisms that reduce adverse selection:

- **Warranties.** An owner of a good car can offer a warranty, which would be very costly for an owner of a lemon to match, since lemons break down often. The warranty is a credible signal of quality (Section 16.4).
- **Independent inspections.** A mechanic or a certification program can check a car's quality before the sale and report it to the buyer, so that a good car sells for close to its full value.

Remark. Other mechanisms include dealer reputation and online ratings (Section 16.7), vehicle-history reports, and lemon laws that force car dealers to honor warranties (Section 16.6.2).

Exercise R14.2. Three bidders compete for a vintage camera. Bidder 1 values it at \$500, Bidder 2 at \$800, and Bidder 3 at \$1,200. Each bidder knows only her own value.

- (a) In a second-price sealed-bid (Vickrey) auction, what is each bidder's dominant strategy? Who wins, and what price does the winner pay?
- (b) In a first-price sealed-bid auction, each bidder shades her bid below her value. Consider that each bidder bids two-thirds of her value (with more rivals, she would shade less, as Section 17.3 explains). Find each bid. Who wins, and at what price?
- (c) Compare the seller's revenue under the two formats. Revenue equivalence (Section 17.4) says that the formats raise the same expected revenue. Why does it not require the two formats to raise the same revenue in every auction? (Hint: redo parts (a) and (b) with Bidder 2's value at \$600 instead of \$800.)

Solution.

Part (a). In a second-price auction, each bidder's dominant strategy is to bid her value (Section 17.3). Her bid decides only whether she wins, while the price she pays is the second-highest bid:

- Bidding above her value only matters when she would win at a price above what the camera is worth to her, a loss.
- Bidding below her value only matters when she would lose an auction she would have been glad to win at a price below her value.

The bids are therefore \$500, \$800, and \$1,200. Bidder 3 places the highest bid and wins, paying the second-highest bid, \$800. Her gain is $\$1,200 - \$800 = \$400$.

Part (b). Each bidder bids two-thirds of her value:

$$\text{Bidder 1: } \frac{2}{3} \times \$500 \approx \$333.33,$$

$$\text{Bidder 2: } \frac{2}{3} \times \$800 \approx \$533.33,$$

$$\text{Bidder 3: } \frac{2}{3} \times \$1,200 = \$800.$$

(Bids are rounded to the nearest cent.) Bidder 3 again places the highest bid, wins, and pays her own bid, \$800. Her gain is again \$400.

Part (c). In this example, the seller earns \$800 under both formats, but the tie depends on the particular values: two-thirds of Bidder 3's \$1,200 happens to equal Bidder 2's value of \$800. Changing Bidder 2's value shows this:

- With Bidder 2's value at \$600, the second-price auction raises \$600 (Bidder 3 pays Bidder 2's bid), while the first-price auction still raises \$800 (Bidder 3 pays two-thirds of \$1,200).
- With Bidder 2's value at \$1,000, the second-price auction raises \$1,000, while the first-price auction still raises \$800.

The formats differ in who pays what in any single auction, and either one can raise more. Revenue equivalence is a statement about the *average*: in a first-price auction, the winner shades her bid but pays her own bid, while in a second-price auction she bids her value but pays the second-highest bid, and, averaged over all the values the bidders might have, the two effects cancel (Section 17.4).

Remark. Section 17.4 also lists the conditions behind revenue equivalence: bidders who are alike, whose private values are drawn independently in the same way, and who care only about their expected profit. When these conditions fail (for instance, when bidders are risk averse, values are correlated, or some bidders are stronger), the formats raise different expected revenues, and the choice of format matters.

Exercise R14.3. A factory sells widgets, and its production pollutes the air of a nearby town. The table lists, for each widget produced per day, buyers' marginal benefit (the demand schedule) and the factory's private marginal cost (the supply schedule). Each widget also causes \$6 of harm to the town's residents, who are not part of the market.

Widget	Marginal benefit	Private marginal cost
1st	\$19	\$3
2nd	\$18	\$4
3rd	\$17	\$5
4th	\$16	\$6
5th	\$15	\$7
6th	\$14	\$8
7th	\$13	\$9
8th	\$12	\$10
9th	\$11	\$11
10th	\$10	\$12

- The unregulated market produces every widget whose marginal benefit is at least its private marginal cost. How many widgets does it produce, and at what price?
- Add a column for the social marginal cost of each widget. Find the efficient quantity, where each widget's marginal benefit is at least its social marginal cost, and the price at that quantity.

- (c) Compute the deadweight loss of the unregulated market.
- (d) What corrective (Pigouvian) tax per widget leads the factory to produce the efficient quantity?
- (e) Verify that, with this tax, the factory produces the efficient quantity, and find the tax revenue. Illustrate your answers in a diagram showing the demand schedule, the private and social marginal cost schedules, the two quantities, and the deadweight loss.

Solution.

Part (a). The marginal benefit falls by \$1 and the private marginal cost rises by \$1 with each widget. They are equal at the 9th widget (\$11 each), and the 10th widget costs more (\$12) than buyers value it (\$10). The unregulated market produces 9 widgets a day, at a price of \$11.

Part (b). Each widget's social marginal cost is its private marginal cost plus the \$6 of harm it causes:

Widget	Marginal benefit	Private marginal cost	Social marginal cost
1st	\$19	\$3	\$9
2nd	\$18	\$4	\$10
3rd	\$17	\$5	\$11
4th	\$16	\$6	\$12
5th	\$15	\$7	\$13
6th	\$14	\$8	\$14
7th	\$13	\$9	\$15
8th	\$12	\$10	\$16
9th	\$11	\$11	\$17
10th	\$10	\$12	\$18

Marginal benefit is at least social marginal cost up to the 6th widget, where both equal \$14, and below it from the 7th widget on. The efficient quantity is 6 widgets a day, at a price of \$14. The market overproduces by 3 widgets, as in Section 18.2.

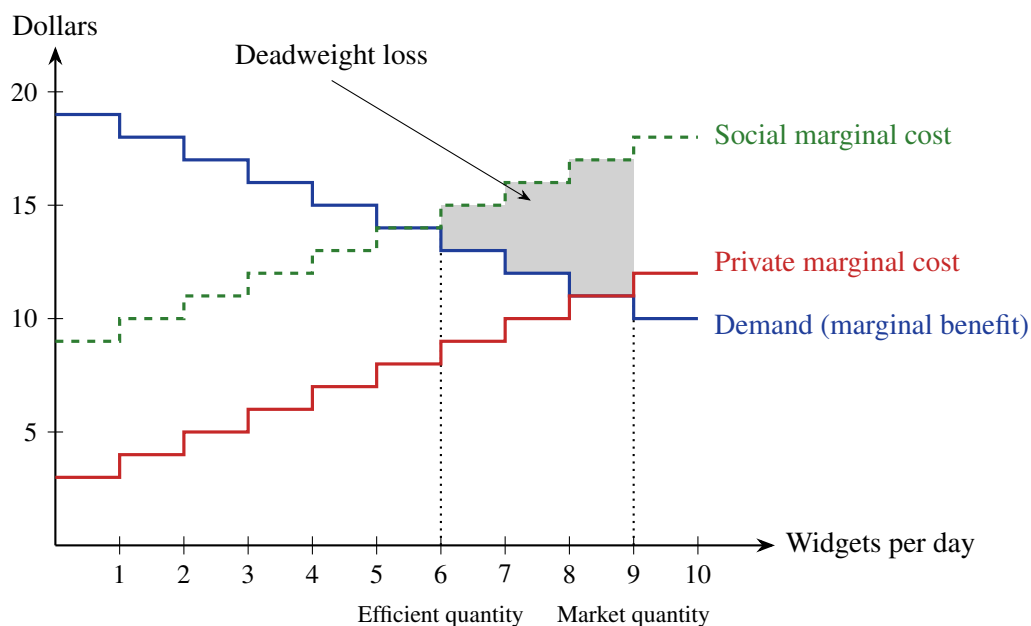
Part (c). The 7th, 8th, and 9th widgets should not be produced, since each costs society more than buyers value it. Adding the amount by which social marginal cost exceeds marginal benefit over these widgets gives the deadweight loss:

$$(\$15 - \$13) + (\$16 - \$12) + (\$17 - \$11) = \$2 + \$4 + \$6 = \$12 \text{ per day.}$$

Part (d). A corrective tax equal to the external harm, \$6 per widget, raises the factory's cost of each widget to its social marginal cost (Section 18.3).

Part (e). With the tax, the factory's cost of each widget is its private marginal cost plus \$6, which is the social marginal cost column of part (b). The 6th widget costs the factory $\$8 + \$6 = \$14$, the same as buyers' marginal benefit, so it is produced; the 7th costs $\$9 + \$6 = \$15$, more than buyers' \$13, so it is not. The factory produces the efficient 6 widgets, buyers pay \$14, and the tax raises $\$6 \times 6 = \$36$ a day.

The figure below draws the three schedules as steps, one step per widget. The shaded bars are the gaps between social marginal cost and marginal benefit for the 7th, 8th, and 9th widgets; their areas add up to the \$12 deadweight loss.



Remark. The tax does not remove all pollution: the 6 widgets still cause $\$6 \times 6 = \$36$ of harm a day. Each of these widgets is worth at least as much to buyers as its full cost to society, so producing them is efficient.