

Review Session Handout #13

Chapter 15: Oligopoly and Imperfect Competition

Introduction to Microeconomics: Markets, Strategy, and Real-World Cases

Felix Muñoz-Garcia, Washington State University

Exercise R13.1. Two identical firms make the same product and compete in quantities (Cournot, Section 15.3): each chooses how much to produce at the same time as the other. The price, in dollars, equals 24 minus the total number of units the two firms sell. Each unit costs \$6 to produce, and there is no fixed cost. (Because each unit costs \$6, the numbers differ from those of the zero-cost example in Section 15.3.)

- (a) Firm 2 might produce 0, 2, 4, 6, or 8 units. For each of these outputs, find Firm 1's best output by comparing its profit (the price minus \$6, times its own output) from producing 4, 5, 6, 7, 8, 9, or 10 units. This list of best outputs is Firm 1's reaction function.
- (b) Firm 2's reaction function is the same as Firm 1's. Find the Nash equilibrium: the pair of outputs in which each firm's output is a best response to the other's.
- (c) Find the equilibrium price and each firm's profit.
- (d) Compare the Cournot outcome with the competitive outcome (where the price equals the cost of \$6 per unit) and with the monopoly outcome (use your answer to part (a) for a rival output of 0). Illustrate the two reaction functions and the Nash equilibrium in a diagram.

Solution.

Part (a). Firm 1's profit equals the price minus \$6, times its own output, where the price is 24 minus the two firms' total output. For example, if Firm 2 produces 4 units and Firm 1 produces 7, total output is 11, the price is \$13, and Firm 1 earns $(13 - 6) \times 7 = \$49$. The table lists Firm 1's profit (in dollars) for each pair of outputs, with the largest profit in each column underlined.

Firm 1's output	Firm 2's output				
	0	2	4	6	8
4	56	48	40	32	24
5	65	55	45	35	<u>25</u>
6	72	60	48	<u>36</u>	24
7	77	63	<u>49</u>	35	21
8	80	<u>64</u>	48	32	16
9	<u>81</u>	63	45	27	9
10	80	60	40	20	0

Firm 1's reaction function is therefore:

Firm 2's output	0	2	4	6	8
Firm 1's best output	9	8	7	6	5

Each extra 2 units from Firm 2 lower Firm 1's best output by 1 unit: when its rival sells more, the price falls, and Firm 1 does best to cut back.

Part (b). Firm 2's reaction function is the same as Firm 1's, so Firm 2's best response to 6 units from Firm 1 is also 6 units. When each firm produces 6 units, each output is a best response to the other's, so the Nash equilibrium is 6 units for each firm. No other pair in the table works: for example, if Firm 2 produced 4 units, Firm 1 would produce 7, but Firm 2's best response to 7 units is not 4.

Part (c). Total output is $6 + 6 = 12$ units, so the price is $24 - 12 = \$12$. Each firm's profit is

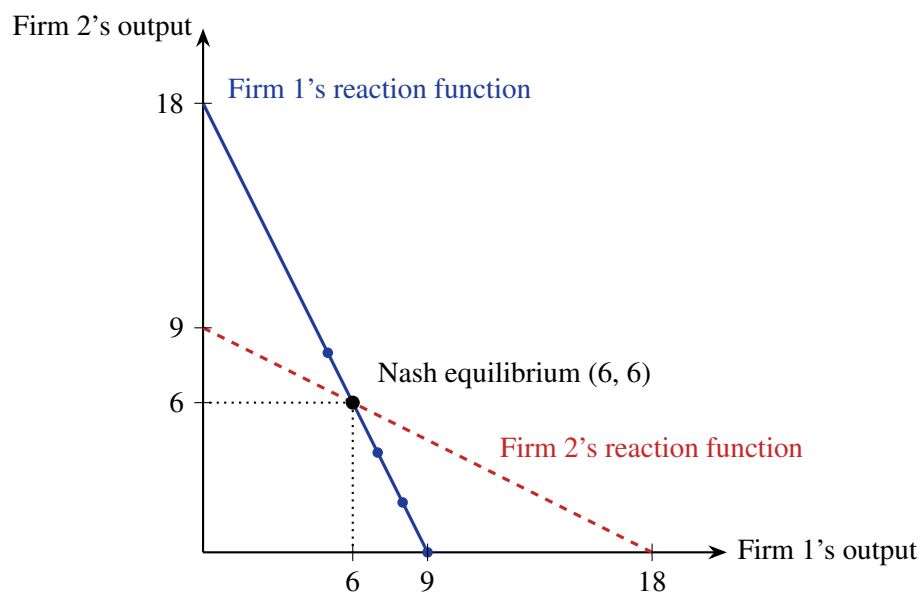
$$(\$12 - \$6) \times 6 = \$36.$$

Part (d). In the competitive outcome, the price equals the cost of \$6 per unit. Since the price is 24 minus total output, total output is $24 - 6 = 18$ units, and profit is zero. In the monopoly outcome, a single firm faces no rival output, so the column for a rival output of 0 in part (a) applies: the monopolist produces 9 units, the price is $24 - 9 = \$15$, and its profit is $(\$15 - \$6) \times 9 = \$81$. If two firms colluded and split the monopoly output equally (Section 15.6), each would produce 4.5 units and earn \$40.50.

Outcome	Total output	Price	Profit per firm	Total profit
Competition	18	\$6	\$0	\$0
Cournot (two firms)	12	\$12	\$36	\$72
Collusion, split equally	9	\$15	\$40.50	\$81
Monopoly (one firm)	9	\$15	\$81	\$81

Cournot output (12 units) lies between the monopoly output (9) and the competitive output (18), and the Cournot price (\$12) lies between the monopoly price (\$15) and the competitive price (\$6). As Section 15.3 explains, with more firms the outcome moves closer to perfect competition.

The figure below plots the two reaction functions. The dots on Firm 1's reaction function are the best outputs of part (a). The two reaction functions cross at the Nash equilibrium, where each firm produces 6 units.



Exercise R13.2. Consider the same market as in Exercise R13.1, but now Firm 1 (the leader) chooses its output first, and Firm 2 (the follower) observes it before choosing its own output (Stackelberg, Section 15.5). Outputs can include half units. The table gives the follower's best output for five outputs of the leader.

Leader's output	6	8	9	10	12
Follower's best output	6	5	4.5	4	3

- Explain why the follower's best outputs are those of Firm 2's reaction function in Exercise R13.1. Check the entry for a leader output of 8 by comparing the follower's profit from producing 4, 5, and 6 units.
- For each leader output in the table, find total output, the price, and the leader's profit. Using backward induction, which of these outputs does the leader choose?
- Find the follower's output, the market price, and each firm's profit.
- Compare the Stackelberg outcome with the Cournot outcome of Exercise R13.1. Who gains and who loses when Firm 1 moves first? Are consumers better or worse off?

Solution.

Part (a). The follower chooses after observing the leader's output, so it takes that output as given, exactly as Firm 2 does when it reacts to Firm 1's output in the Cournot game. Its best output for each leader output is therefore given by the same reaction function. For a leader output of 8:

- Producing 4 units: total output 12, price \$12, profit $(\$12 - \$6) \times 4 = \$24$.
- Producing 5 units: total output 13, price \$11, profit $(\$11 - \$6) \times 5 = \$25$.
- Producing 6 units: total output 14, price \$10, profit $(\$10 - \$6) \times 6 = \$24$.

The follower's best output is 5 units, as the table says.

Part (b). The leader looks ahead to the follower's response:

Leader's output	Follower's output	Total output	Price	Leader's profit	Follower's profit
6	6	12	\$12.00	\$36.00	\$36.00
8	5	13	\$11.00	\$40.00	\$25.00
9	4.5	13.5	\$10.50	\$40.50	\$20.25
10	4	14	\$10.00	\$40.00	\$16.00
12	3	15	\$9.00	\$36.00	\$9.00

By backward induction, the leader chooses the output that gives it the highest profit once the follower's response is taken into account: 9 units, which earn it \$40.50.

Part (c). The follower produces 4.5 units, total output is 13.5 units, and the price is $24 - 13.5 = \$10.50$. Profits are

$$\text{Leader: } (\$10.50 - \$6) \times 9 = \$40.50,$$

$$\text{Follower: } (\$10.50 - \$6) \times 4.5 = \$20.25.$$

Part (d). Compared with the Cournot outcome of Exercise R13.1 (6 units each, a price of \$12, and \$36 for each firm):

Outcome	Firm 1's output	Firm 2's output	Price	Firm 1's profit	Firm 2's profit
Cournot	6	6	\$12	\$36	\$36
Stackelberg	9	4.5	\$10.50	\$40.50	\$20.25

- The leader gains: its output rises from 6 to 9 units, and its profit rises from \$36 to \$40.50.
- The follower loses: its output falls from 6 to 4.5 units, and its profit falls from \$36 to \$20.25.
- Consumers gain: total output rises from 12 to 13.5 units, and the price falls from \$12 to \$10.50.

Remark. The leader's advantage comes from committing to a large output before the follower acts (Section 15.5). Once the leader's 9 units are on the market, the follower does best to produce less, which is the first-mover advantage that Chapter 14 traces to commitment.

Exercise R13.3. Two firms sell identical bottles of water, each at a cost of \$1 per bottle, with no fixed costs. At a price of \$1, buyers purchase 90 bottles a day, and each \$1 increase in the price reduces purchases by 10 bottles (so 60 bottles sell at \$4, and none sell at \$10).

- In Bertrand competition (Section 15.4), what price does each firm charge in equilibrium?
- What is each firm's profit?
- Under Cournot competition, each firm would sell 30 bottles a day, and a monopolist would sell 45 bottles a day. Find the price and the total profit in each case, and compare them with the Bertrand outcome.
- Explain the Bertrand paradox. Why are just two firms enough to produce the competitive outcome?
- Name two real-world features that weaken the Bertrand result, so that firms can charge a price above marginal cost.

Solution.

Part (a). With identical bottles and the same cost of \$1 per bottle, the only Nash equilibrium is for both firms to charge \$1, the marginal cost.

Part (b). At a price of \$1, buyers purchase 90 bottles a day, and each firm earns \$1 minus \$1, or zero, on every bottle it sells. Each firm's profit is \$0.

Part (c). Using the description of demand:

- Cournot: the two firms sell $30 + 30 = 60$ bottles, which sell at \$4. Each firm earns $(\$4 - \$1) \times 30 = \$90$, so total profit is \$180.
- Monopoly: 45 bottles sell at \$5.50, since 45 bottles are 45 fewer than the 90 that sell at \$1, and each \$1 increase in the price reduces purchases by 10 bottles, so the price is \$4.50 above \$1. Profit is $(\$5.50 - \$1) \times 45 = \$202.50$.

Outcome	Bottles sold	Price	Total profit
Bertrand	90	\$1.00	\$0.00
Cournot	60	\$4.00	\$180.00
Monopoly	45	\$5.50	\$202.50

Bertrand competition gives the lowest price, the most bottles sold, and zero profit. The Cournot outcome lies between the Bertrand and monopoly outcomes.

Part (d). With identical products, buyers purchase from whichever firm charges less. If Firm 1 charges any price above \$1, Firm 2 can charge a penny less and serve all buyers, as Section 15.4 explains. Firm 1 would then undercut Firm 2 in turn, and the undercutting continues until both firms charge \$1, the marginal cost. The paradox is that just two firms are enough to remove all market power and reproduce the competitive outcome.

Part (e). Section 15.4 names three such features (any two answer the question):

- Product differentiation. When the two products are not identical, each firm faces a downward-sloping demand curve and can charge above marginal cost without losing all of its customers (Section 15.7).
- Capacity limits. If the firm that undercuts its rival cannot serve the whole market, the rival keeps some customers and can also charge above marginal cost.
- Repeated interaction. Firms that meet week after week can sustain high prices through the threat of a future price war, the tacit collusion of Section 14.5.1.

Remark. Laboratory evidence points the same way. In the Bertrand experiments of Dufwenberg and Gneezy (2000), summarized in Table 15.1 of the book, prices stayed above cost with two firms and fell close to it only with three or four.