

## Review Session Handout #12

### Chapter 14: Game Theory II: Sequential and Repeated Games

*Introduction to Microeconomics: Markets, Strategy, and Real-World Cases*

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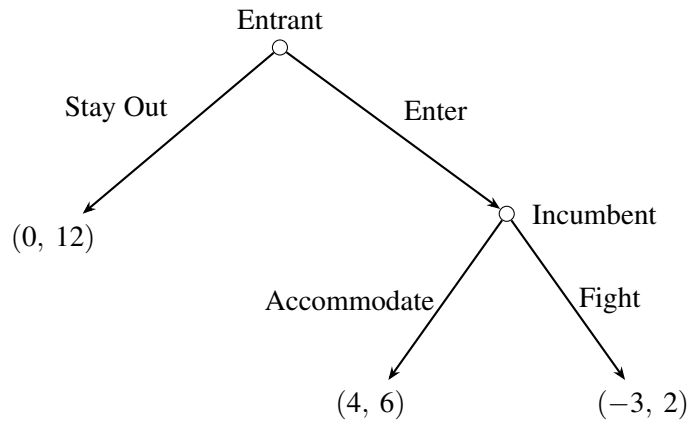
#### Solution to Exercise R12.1

**Exercise R12.1.** A discount grocery chain (the entrant) is deciding whether to open a store in a town served by a single supermarket (the incumbent). The entrant moves first and chooses *Enter* or *Stay Out*. If it stays out, the incumbent remains the only supermarket in town. If it enters, the incumbent observes this and chooses to *Accommodate* (share the market peacefully) or *Fight* (start a price war). The payoffs, in hundreds of thousands of dollars per year, are listed as (entrant, incumbent):

- The entrant stays out:  $(0, 12)$ .
- The entrant enters and the incumbent accommodates:  $(4, 6)$ .
- The entrant enters and the incumbent fights:  $(-3, 2)$ .

- Draw the game tree.
- Use backward induction: if the entrant enters, does the incumbent accommodate or fight?
- Given your answer to part (b), does the entrant enter? State the subgame-perfect equilibrium.
- Before the entrant decides, the incumbent announces that it will start a price war if the entrant opens a store. Is this a credible threat? Explain.
- Consider now that, before the entrant moves, the incumbent can sign long-term contracts with its suppliers for large volumes. The contracts cost the incumbent 1, and the extra volume is useful only in a price war, where it raises the incumbent's payoff from fighting by 5 (before the cost of 1 is subtracted). The entrant observes whether the incumbent signs. List the payoffs at the end of each path after the incumbent signs, and solve the game by backward induction. Does the incumbent sign the contracts, and does the entrant enter?

**Part (a).** The entrant moves first, at the top of the tree. Staying out ends the game; entering leads to the incumbent's decision. Payoffs are listed as (entrant, incumbent).



**Part (b).** We start at the last decision. If the entrant has entered, the incumbent earns 6 by accommodating and 2 by fighting. Since 6 is larger than 2, the incumbent accommodates.

**Part (c).** The entrant anticipates the incumbent's response:

- If it enters, the incumbent accommodates, and the entrant earns 4.
- If it stays out, the entrant earns 0.

Since 4 is larger than 0, the entrant enters. The subgame-perfect equilibrium is:

- Entrant: Enter.
- Incumbent: Accommodate if the entrant enters.

The outcome is that the entrant enters and the incumbent accommodates, with payoffs (4, 6). Each player acts optimally at its own decision point, as Section 14.3 requires of a subgame-perfect equilibrium.

**Part (d).** No. Once the entrant has opened its store, a price war gives the incumbent 2, while accommodating gives it 6. A rational incumbent would not carry out the threat, so it is a *noncredible threat* (Section 14.4), and the entrant ignores it and enters. The announcement does not change the payoffs at the end of the tree, so it does not change the incumbent's choice when that choice arrives.

**Part (e).** If the incumbent signs, its payoffs fall by the cost of 1 on every path, and its payoff from fighting also rises by 5. The payoffs (entrant, incumbent) at the end of each path become:

- The entrant stays out: (0, 11), since  $12 - 1 = 11$ .
- The entrant enters and the incumbent accommodates: (4, 5), since  $6 - 1 = 5$ .
- The entrant enters and the incumbent fights: (-3, 6), since  $2 + 5 - 1 = 6$ .

If the incumbent does not sign, the payoffs are those of parts (a) to (c).

We solve by backward induction:

- After signing and entry, the incumbent earns 6 by fighting and 5 by accommodating, so it fights. The threat to fight is now credible.

- After signing, the entrant foresees a price war that would give it  $-3$ , less than the 0 it earns by staying out, so it stays out. The incumbent earns 11.
- Without the contracts, parts (b) and (c) apply: the entrant enters, the incumbent accommodates, and the incumbent earns 6.

Since 11 is larger than 6, the incumbent signs the contracts, and the entrant stays out. The price war never happens: the contracts deter entry without a shot being fired. As in Figure 14.2, a costly commitment that limits the incumbent's own options makes its threat believable and leaves it better off.

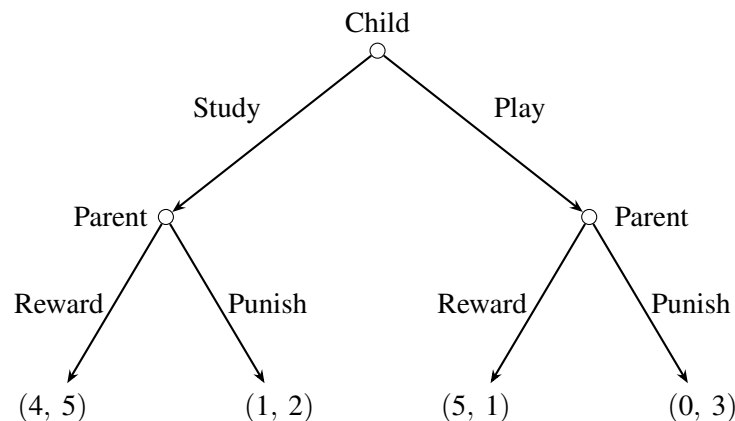
## Solution to Exercise R12.2

**Exercise R12.2.** A parent and a child play a homework game. The child moves first and chooses to *Study* or *Play*. After observing the child's choice, the parent responds with *Reward* or *Punish*. The payoffs are listed as (child, parent):

- (Study, Reward): (4, 5).
- (Study, Punish): (1, 2).
- (Play, Reward): (5, 1).
- (Play, Punish): (0, 3).

- Draw the game tree.
- Find the subgame-perfect equilibrium using backward induction.
- The parent threatens: "If you play, I will punish you." Is this threat credible? Examine the parent's payoffs after the child chooses Play.
- Given your answer to part (c), what does the child choose? Explain why a noncredible threat would fail to change the child's behavior.

**Part (a).** The child moves first, and the parent responds after observing the child's choice. Payoffs are listed as (child, parent).



**Part (b).** We start with the parent's choices.

- After Study, the parent earns 5 from Reward and 2 from Punish, so it rewards.
- After Play, the parent earns 1 from Reward and 3 from Punish, so it punishes.

The child anticipates these responses:

- Studying leads the parent to reward, giving the child 4.
- Playing leads the parent to punish, giving the child 0.

Since 4 is larger than 0, the child studies. The subgame-perfect equilibrium is:

- Child: Study.
- Parent: Reward after Study; Punish after Play.

The outcome is (Study, Reward), with payoffs (4, 5).

**Part (c).** After the child chooses Play, the parent earns 1 from Reward and 3 from Punish. Since 3 is larger than 1, the parent prefers to punish once the child has played, so the threat is credible: carrying it out is in the parent's own interest when the time comes.

*Remark.* This differs from the entry game of Figure 14.1, where fighting after entry earns the incumbent  $-2$  against 5 from accommodating, and from Exercise R12.1, where fighting earns 2 against 6. In those games, carrying out the threat hurts the player who makes it, so the threat is not credible. Here, the parent's threat matches the parent's own incentive after Play.

**Part (d).** Since the threat is credible, the child anticipates that playing leads to punishment and a payoff of 0, while studying leads to a reward and a payoff of 4. The child studies.

A noncredible threat would not have this effect. Consider, for example, that the parent earned 4 instead of 1 from rewarding the child after Play. After Play, the parent would then reward (4 against 3), so the child would expect a payoff of 5 from playing, more than the 4 from studying, and would play. The parent could still announce the punishment, but the child, reasoning by backward induction, would know that the parent would not carry it out. A threat changes behavior only if the player who makes it would want to carry it out when the time comes.

### Solution to Exercise R12.3

**Exercise R12.3.** Two firms play a prisoner's dilemma. The payoff matrix lists their profits per period (in thousands of dollars), with Firm 1's payoff first in each cell.

|        |           | Firm 2    |        |
|--------|-----------|-----------|--------|
|        |           | Cooperate | Defect |
| Firm 1 | Cooperate | 8, 8      | 1, 14  |
|        | Defect    | 14, 1     | 4, 4   |

- (a) What is the Nash equilibrium if this game is played exactly once?
- (b) Consider that the game is played exactly 3 times, and both firms know it. Use backward induction to explain the equilibrium outcome. How much does each firm earn in total?
- (c) Consider now that the game is repeated indefinitely (the firms never know for sure when it will end), and each firm uses grim trigger: it cooperates until its rival defects, and then defects in every later period. The firms expect the game to last 2 periods in total, counting the current one. Ignoring any other discounting of future profits, compare a firm's expected total profit from always cooperating with its expected total profit from defecting in the current period (and in every later period, once its rival punishes it). Repeat the comparison for an expected 5 periods. In which case can cooperation survive?
- (d) What is the smallest expected number of periods at which always cooperating earns at least as much as defecting? Interpret your answer: why must the firms value the future enough for cooperation to survive?

**Part (a).** We check Firm 1's best responses:

- If Firm 2 cooperates, Firm 1 earns 8 from cooperating and 14 from defecting, so its best response is Defect.
- If Firm 2 defects, Firm 1 earns 1 from cooperating and 4 from defecting, so its best response is again Defect.

Defect is a dominant strategy for Firm 1, and, since the matrix is symmetric, also for Firm 2. The only Nash equilibrium is (Defect, Defect), with payoffs of 4 for each firm, even though both would earn 8 if both cooperated.

**Part (b).** We apply backward induction, starting from the last period.

- Period 3 (the last). No future period is left to protect, so the firms face the one-shot game of part (a). Both defect.
- Period 2. Both firms know that period 3 ends in (Defect, Defect) no matter what happens in period 2. Cooperating in period 2 cannot earn any reward later, so the firms again face the one-shot game. Both defect.
- Period 1. The same reasoning applies: both firms defect.

Cooperation unravels from the last period back to the first, as in Section 14.5. Both firms defect in every period, and each earns 4 per period, or 12 in total over the three periods.

**Part (c).** Under grim trigger, a firm that defects in the current period earns 14 now, but its rival then defects in every later period, and the best the defector can do in those periods is to defect too, earning 4 per period.

*Expected 2 periods.* The firm's expected totals are

$$\text{Always cooperating: } 8 + 8 = 16,$$

$$\text{Defecting now: } 14 + 4 = 18.$$

Defecting earns more (18 against 16), so cooperation cannot survive.

*Expected 5 periods.* The firm's expected totals are

$$\text{Always cooperating: } 8 \times 5 = 40,$$

$$\text{Defecting now: } 14 + 4 \times 4 = 30.$$

Always cooperating earns more (40 against 30), so cooperation survives.

**Part (d).** Defecting raises the firm's profit in the current period by 6 (14 instead of 8), but lowers it by 4 in every later period (4 instead of 8). Cooperation survives when the expected number of later periods is at least  $\frac{6}{4} = 1.5$ , that is, when the game is expected to last at least 2.5 periods in total, counting the current one. At exactly 2.5 periods, the two totals are equal:

$$\text{Always cooperating: } 8 \times 2.5 = 20,$$

$$\text{Defecting now: } 14 + 4 \times 1.5 = 20.$$

Below 2.5 periods, defecting earns more; above 2.5 periods, always cooperating earns more.

Intuitively, the gain from defecting comes now, while the loss comes from the cooperation that grim trigger withdraws in later periods. When the firms expect the game to last longer, there is more future cooperation to lose, so the threat of punishment is stronger. This is what Section 14.5 means by players who value the future enough. Dal Bó (2005) found the same pattern in the laboratory: a higher chance that another round would follow made opportunistic play less common.

*Remark.* The cutoff depends on how large the one-time gain from defecting (6) is relative to the loss in each later period (4). A larger temptation, or a milder punishment, raises the expected number of periods that cooperation needs.