

Review Session Handout #11

Chapter 13: Game Theory I: Simultaneous-Move Games

Introduction to Microeconomics: Markets, Strategy, and Real-World Cases

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Solution to Exercise R11.1

Exercise R11.1. Two coffee shops, Brew and Grind, each choose, at the same time, between *High* and *Low* advertising. The payoff matrix below lists their daily profits (in hundreds of dollars), with Brew's payoff first in each cell.

		Grind	
		High	Low
Brew	High	3, 3	7, 1
	Low	1, 7	5, 5

- (a) Explain what each number in the matrix means. Whose payoff comes first in each cell?
- (b) Does Brew have a dominant strategy? Does Grind?
- (c) Find all Nash equilibria.
- (d) Is this a prisoner's dilemma? Explain by comparing the Nash equilibrium outcome with the outcome (Low, Low).

Part (a). Each row is one of Brew's strategies, each column is one of Grind's, and each cell lists the two shops' daily profits for that combination of choices. As in Section 13.2, the first number is the row player's payoff (Brew) and the second is the column player's (Grind). For example, the cell (High, Low) says that, if Brew advertises heavily and Grind does not, Brew earns \$700 a day and Grind earns \$100.

Part (b). We check Brew's best responses:

- If Grind chooses High, Brew earns 3 from High and 1 from Low, so its best response is High.
- If Grind chooses Low, Brew earns 7 from High and 5 from Low, so its best response is again High.

High is Brew's best response to both of Grind's choices, so High is a dominant strategy for Brew. Because the matrix is symmetric, the same comparisons hold for Grind (3 against 1, and 7 against 5), so High is also a dominant strategy for Grind.

Part (c). Since each shop has a dominant strategy, the only Nash equilibrium is (High, High), with payoffs of 3 for each shop. Given that the other shop advertises heavily, neither shop gains by switching to Low on its own (it would earn 1 instead of 3).

Part (d). Yes, this is a prisoner's dilemma. Each shop has a dominant strategy (High), yet both shops are better off at (Low, Low), where each earns 5, than at the equilibrium (High, High), where each earns 3. Both would gain if they could commit to Low advertising, but no matter what the rival does, each shop earns more by choosing High. The Nash equilibrium, in which each shop plays its dominant strategy, therefore leaves both shops worse off than if both had advertised less.

The payoff matrix below has each player's best responses underlined, following the best-response method of Section 13.4.1.

		Grind	
		High	Low
Brew	High	<u>3</u> <u>3</u> Nash equilibrium	<u>7</u> 1
	Low	1 <u>7</u>	5 5

Remark. The only cell in which both payoffs are underlined is (High, High), so it is the Nash equilibrium, even though both shops would earn more at (Low, Low).

Solution to Exercise R11.2

Exercise R11.2. Two technology firms choose, at the same time, which format standard to adopt: *Blu-ray* or *HD-DVD*. The payoff matrix lists their profits, with Firm 1's payoff first in each cell.

		Firm 2	
		Blu-ray	HD-DVD
Firm 1	Blu-ray	4, 4	0, 0
	HD-DVD	0, 0	3, 3

- Find all pure-strategy Nash equilibria (there are two).
- Explain why this is a coordination game. Why is it not a prisoner's dilemma?
- Which equilibrium do both firms prefer?
- Why might the firms end up at the worse equilibrium?
- Consider that Firm 1 believes that there is a one-in-four chance that Firm 2 adopts Blu-ray. Using expected values (Chapter 7), which format gives Firm 1 the higher expected payoff? Above what chance that Firm 2 adopts Blu-ray does Blu-ray give Firm 1 the higher expected payoff? Relate your answer to part (d).

Part (a). We find each player's best responses:

- If Firm 2 chooses Blu-ray, Firm 1 earns 4 from Blu-ray and 0 from HD-DVD, so its best response is Blu-ray.
- If Firm 2 chooses HD-DVD, Firm 1 earns 0 from Blu-ray and 3 from HD-DVD, so its best response is HD-DVD.

The matrix is symmetric, so Firm 2's best responses are the same as Firm 1's. Both best responses land in two cells, so there are two pure-strategy Nash equilibria:

- (Blu-ray, Blu-ray), with payoffs of 4 for each firm.
- (HD-DVD, HD-DVD), with payoffs of 3 for each firm.

In each equilibrium, both firms adopt the same standard, and neither firm gains by switching on its own (it would earn 0).

Part (b). In a coordination game, players do better when their choices align (Section 13.5), and here both firms earn 0 whenever they adopt different standards. In a prisoner's dilemma, each player has a dominant strategy, and the equilibrium leaves both worse off than another cell. In this game, neither firm has a dominant strategy: each firm's best format depends on the format the other firm adopts. The firms' problem is to *coordinate* on the same standard. Once both firms have adopted the same standard, neither firm is tempted to deviate, unlike the players in a prisoner's dilemma.

Part (c). Both firms prefer (Blu-ray, Blu-ray), where each earns 4, to (HD-DVD, HD-DVD), where each earns 3. Both firms are better off at (Blu-ray, Blu-ray), so it is the equilibrium both firms prefer.

Part (d). The firms can end up at (HD-DVD, HD-DVD) if each firm *expects* the other to choose HD-DVD. Once both expect HD-DVD, neither firm gains by switching to Blu-ray on its own, so the worse equilibrium is self-sustaining. As Section 13.5 explains, which equilibrium the players reach may depend on history, expectations, or a focal point: an earlier product launched in HD-DVD, or a large stock of HD-DVD players already in homes, can lead both firms to expect HD-DVD.

Part (e). With a one-in-four chance that Firm 2 adopts Blu-ray (and a three-in-four chance that it adopts HD-DVD), Firm 1's expected payoffs are

$$\begin{aligned}\text{Blu-ray: } & \frac{1}{4} \times 4 + \frac{3}{4} \times 0 = 1, \\ \text{HD-DVD: } & \frac{1}{4} \times 0 + \frac{3}{4} \times 3 = 2.25.\end{aligned}$$

HD-DVD gives Firm 1 the higher expected payoff, even though Firm 1 would earn more if both firms adopted Blu-ray.

Blu-ray earns Firm 1 a payoff of 4 each time Firm 2 also adopts Blu-ray, while HD-DVD earns 3 each time Firm 2 adopts HD-DVD. The two expected payoffs are equal when Firm 2 adopts Blu-ray with a chance of

$\frac{3}{7}$ (about 43 percent):

$$\text{Blu-ray: } \frac{3}{7} \times 4 = \frac{12}{7},$$

$$\text{HD-DVD: } \frac{4}{7} \times 3 = \frac{12}{7}.$$

So Blu-ray gives Firm 1 the higher expected payoff only when the chance that Firm 2 adopts Blu-ray is above $\frac{3}{7}$; at exactly $\frac{3}{7}$, the two formats give the same expected payoff.

This is the trap of part (d). Both firms prefer Blu-ray, but if each believes that the other is unlikely to adopt it (a chance below $\frac{3}{7}$), each does best to adopt HD-DVD, and those beliefs turn out to be correct.

Solution to Exercise R11.3

Exercise R11.3. Two firms decide, at the same time, whether to *Enter* or *Stay Out* of a new market. If both firms enter, they split the market and each earns \$1 million. If only one firm enters, it earns \$5 million. A firm that stays out earns \$2 million from its existing business. The payoff matrix lists the payoffs (in millions of dollars), with Firm 1's payoff first in each cell.

		Firm 2	
		Enter	Stay Out
Firm 1	Enter	1, 1	5, 2
	Stay Out	2, 5	2, 2

- (a) Using the description above, explain how each cell of the matrix is built. Whose payoff comes first?
- (b) Does either firm have a dominant strategy?
- (c) Find all pure-strategy Nash equilibria.
- (d) Consider that each firm believes that there is a one-in-two chance that its rival enters. Using expected values (Chapter 7), compare Firm 1's expected payoff from entering with its expected payoff from staying out. If both firms act on this belief, what happens, and what does each firm earn?
- (e) Explain the strategic tension in this game. In what sense does it resemble a game of chicken (Section 13.5.1)?

Part (a). The first number in each cell is Firm 1's payoff (the row player) and the second is Firm 2's (the column player).

- (Enter, Enter): both firms split the new market, so each earns 1.
- (Enter, Stay Out): Firm 1 enters alone and earns 5, while Firm 2 keeps its existing business and earns 2. The cell (Stay Out, Enter) reverses the roles.
- (Stay Out, Stay Out): neither firm enters, so each earns 2 from its existing business.

Part (b). We check Firm 1's best responses:

- If Firm 2 enters, Firm 1 earns 1 from entering and 2 from staying out, so its best response is Stay Out.
- If Firm 2 stays out, Firm 1 earns 5 from entering and 2 from staying out, so its best response is Enter.

Firm 1's best response depends on Firm 2's choice, so Firm 1 does not have a dominant strategy. The matrix is symmetric, so Firm 2 does not have one either.

Part (c). Using the best responses of part (b):

- (Enter, Stay Out): Enter is Firm 1's best response to Stay Out, and Stay Out is Firm 2's best response to Enter. This is a Nash equilibrium, with payoffs of 5 for Firm 1 and 2 for Firm 2.
- (Stay Out, Enter): by symmetry, this is also a Nash equilibrium, with payoffs of 2 for Firm 1 and 5 for Firm 2.
- (Enter, Enter) is not a Nash equilibrium: each firm would rather stay out (2 instead of 1).
- (Stay Out, Stay Out) is not a Nash equilibrium: each firm would rather enter (5 instead of 2).

The game has two pure-strategy Nash equilibria, (Enter, Stay Out) and (Stay Out, Enter). In each, exactly one firm enters the market.

Part (d). With a one-in-two chance that Firm 2 enters, Firm 1's expected payoffs (in millions of dollars) are

$$\begin{aligned}\text{Enter: } & \frac{1}{2} \times 1 + \frac{1}{2} \times 5 = 3, \\ \text{Stay Out: } & \frac{1}{2} \times 2 + \frac{1}{2} \times 2 = 2.\end{aligned}$$

Entering gives Firm 1 the higher expected payoff, \$3 million against \$2 million. Firm 2 holds the same belief about Firm 1 and makes the same comparison, so it also enters. Both firms then enter, and each earns only \$1 million: less than the \$3 million each expected, and less than the \$2 million each would have earned by staying out.

Part (e). Each firm would like to be the only entrant (earning 5), but if both enter, each earns less than if it had stayed out (1 rather than 2). Each firm therefore wants the *other* to back down and stay out, and neither firm has a dominant strategy. This is the structure of chicken in Section 13.5.1: two drivers speed toward each other, each hoping the other swerves, and the worst outcome is the one in which neither backs down. In the two pure-strategy equilibria, one firm backs down and stays out. Part (d) shows the risk: if each firm expects the other to back down often enough, both enter and both end up worse off.

Remark. Although the game is symmetric, each pure-strategy equilibrium is asymmetric: one firm must stay out, and the game gives no way to decide which one.