

Review Session Handout #6

Chapter 7: Choice under Risk and Uncertainty

Introduction to Microeconomics: Markets, Strategy, and Real-World Cases

Felix Muñoz-Garcia, Washington State University

Whenever an exercise below describes a person as risk averse, her utility from each level of wealth is given by the following table (for Sam in Exercise R6.2, wealth is his income for the season). Each extra dollar adds less utility than the one before: the first \$1,600 adds 4 units of utility, while the \$14,400 between \$25,600 and \$40,000 adds the same 4 units.

Wealth	\$0	\$1,600	\$3,600	\$6,400	\$10,000	\$14,400	\$19,600	\$25,600	\$32,400	\$40,000
Utility	0	4	6	8	10	12	14	16	18	20

Exercise R6.1. An investor has \$6,400 in savings and considers two options.

- Option A: Keep the money in the bank, a certain payoff of \$6,400.
- Option B: Invest in a stock that returns \$10,000 with probability 0.6 or \$1,600 with probability 0.4.

The investor is risk averse, with the utility in the table above.

Part (a). Compute the expected value of each option.

Option A pays \$6,400 for certain, so its expected value is \$6,400. Weighting each outcome of Option B by its probability, its expected value is

$$0.6 \times \$10,000 + 0.4 \times \$1,600 = \$6,000 + \$640 = \$6,640.$$

Option B has the higher expected value, by \$240.

Part (b). Using the table, compute the expected utility of each option.

From the table, a sure \$6,400 yields utility 8, so the expected utility of Option A is 8. Option B yields utility 10 (from \$10,000) with probability 0.6 and utility 4 (from \$1,600) with probability 0.4, so its expected utility is

$$0.6 \times 10 + 0.4 \times 4 = 6 + 1.6 = 7.6.$$

Part (c). Which option does the investor choose?

Since Option A yields expected utility 8 and Option B only 7.6, the investor chooses Option A, the safe option, even though Option B has the higher expected value.

Part (d). Is the investor risk averse, risk neutral, or risk loving?

The investor is risk averse: she prefers a sure \$6,400 to a risky prospect whose expected value is even higher. The table shows the diminishing marginal utility of wealth behind this choice: the \$4,800 between \$1,600 and \$6,400 adds 4 units of utility, while the \$3,600 between \$6,400 and \$10,000 adds only 2. The utility she loses if the stock does badly is larger than the utility she gains if it does well.

Remark. A risk-neutral investor cares only about expected value, so she would choose Option B (\$6,640 against \$6,400). The two choices differ because the risk-averse investor gives up \$240 of expected value in exchange for certainty.

Exercise R6.2. Sam sells handmade furniture at an outdoor market, and his income depends on the weather.

- Dry season (probability 0.6): income of \$40,000.
- Rainy season (probability 0.4): many market days are canceled, and his income falls to \$10,000.

Sam is risk averse, with the utility in the table above.

Part (a). Compute Sam's expected income.

Weighting each income by its probability,

$$0.6 \times \$40,000 + 0.4 \times \$10,000 = \$24,000 + \$4,000 = \$28,000.$$

Part (b). Compute Sam's expected utility.

From the table, \$40,000 yields utility 20 and \$10,000 yields utility 10, so his expected utility is

$$0.6 \times 20 + 0.4 \times 10 = 12 + 4 = 16.$$

Part (c). Find the certainty equivalent, that is, the sure income that gives Sam the same utility as the risky season.

Reading the table, a sure income of \$25,600 yields utility 16, the same as his expected utility. His certainty equivalent is therefore \$25,600: Sam would trade the risky season for a guaranteed \$25,600.

Part (d). Compute the risk premium.

The risk premium is the expected value of the risky season minus its certainty equivalent:

$$\$28,000 - \$25,600 = \$2,400.$$

It is the amount of expected income Sam would willingly give up to avoid the risk.

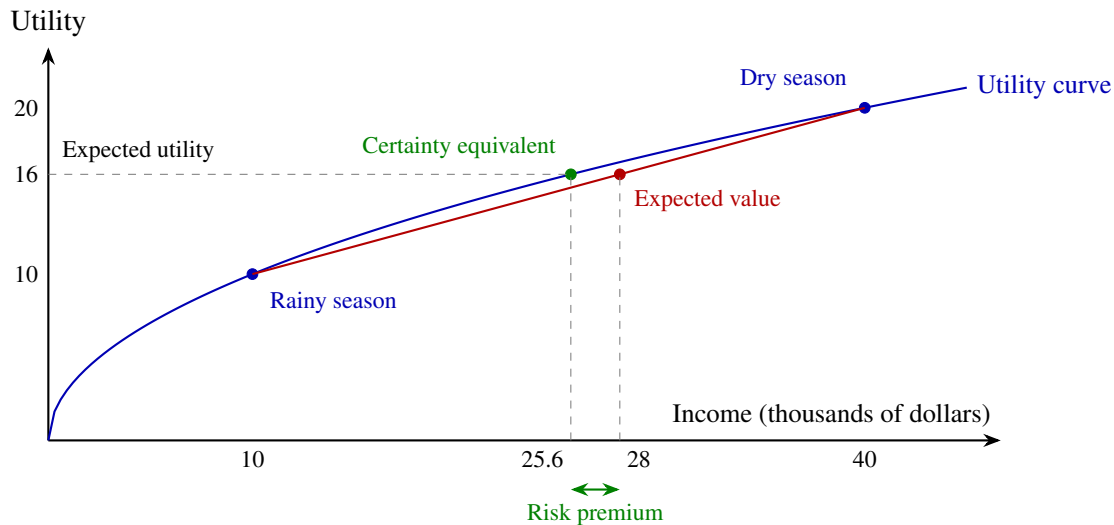
Part (e). An insurance company offers a policy that pays Sam \$30,000 in a rainy season in exchange for a premium of \$14,000. With the policy, his income is $\$40,000 - \$14,000 = \$26,000$ in either season. Should he buy the policy?

Sam's expected loss is $0.4 \times \$30,000 = \$12,000$. A risk-averse person pays up to her expected loss plus her risk premium to be rid of the risk (Section 7.5), so the highest premium Sam would pay is

$$\$12,000 + \$2,400 = \$14,400.$$

This is also $\$40,000 - \$25,600$, the premium that would leave him with exactly his certainty equivalent. Since the \$14,000 premium is below \$14,400, Sam should buy the policy. Equivalently, the sure \$26,000 he keeps with the policy exceeds his certainty equivalent of \$25,600.

The following figure shows Sam's certainty equivalent and risk premium on his utility curve.



Remark. The risk premium alone (\$2,400) is the most Sam would pay *above* his expected loss. The insurer collects \$14,000 per policy while expecting to pay out \$12,000, and Sam still gains, so both sides are better off. At a premium of \$15,000, above \$14,400, Sam would keep only \$25,000, less than his certainty equivalent, and he would not buy.

Exercise R6.3. An entrepreneur chooses among three projects.

- Project A: Returns \$19,600 with probability 0.5 or \$0 with probability 0.5.
- Project B: Returns \$10,000 with certainty.
- Project C: Returns \$40,000 with probability 0.25 or \$0 with probability 0.75.

At first, the entrepreneur is risk neutral: she cares only about expected value.

Part (a). Compute the expected value of each project.

Weighting each outcome by its probability:

- Project A: $0.5 \times \$19,600 + 0.5 \times \$0 = \$9,800$.
- Project B: \$10,000.
- Project C: $0.25 \times \$40,000 + 0.75 \times \$0 = \$10,000$.

Part (b). Which project does the risk-neutral entrepreneur choose?

Projects B and C tie at \$10,000, and she prefers either one to Project A (\$9,800). The risk-neutral entrepreneur is indifferent between Projects B and C.

Part (c). Now consider that the entrepreneur is risk averse, with the utility in the table above. Compute the expected utility of each project and determine her choice.

From the table, \$19,600 yields utility 14, \$10,000 yields 10, \$40,000 yields 20, and \$0 yields 0. The expected utilities are:

- Project A: $0.5 \times 14 + 0.5 \times 0 = 7$.
- Project B: 10.
- Project C: $0.25 \times 20 + 0.75 \times 0 = 5$.

The ranking is Project B (10), then Project A (7), then Project C (5). The risk-averse entrepreneur chooses Project B.

Part (d). Explain why risk attitude changes the ranking of the projects.

Under risk neutrality, only expected value matters, so Projects B and C, with the same expected value of \$10,000, tie. A risk-averse person also cares about the spread of outcomes. Project B pays \$10,000 with no spread at all, while Project C is a large gamble between \$40,000 and nothing. For a risk-averse person, the expected utility of a gamble is below the utility of a sure amount equal to its expected value, and the gap is zero only for a sure thing. A sure \$10,000 yields utility 10, while Project C, with the same expected value, yields expected utility of only 5. Risk aversion therefore breaks the tie in favor of the safe project and pushes the riskiest project, C, to last place.

Remark. Project A has a lower expected value than Project C (\$9,800 against \$10,000), yet it ranks higher under risk aversion. Its outcomes are less spread out: both projects can end with nothing, but Project C does so with probability 0.75 and Project A with probability 0.5. Risk aversion raises the cost of a high probability of receiving nothing.